

# Surfaces

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November 14, 2014

# Homeomorphism

## Definition

A homeomorphism between two geometric figures  $X$  and  $Y$  is a **continuous bijective** map  $f: X \rightarrow Y$  such that its inverse  $f^{-1}: Y \rightarrow X$  is **also continuous**.

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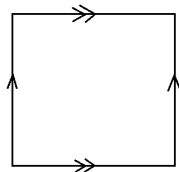
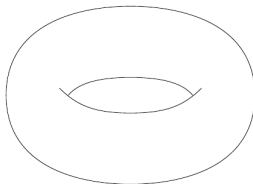
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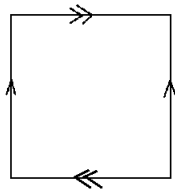
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Sphere: any surface homeomorphic to  $\{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 = 1\}$ .

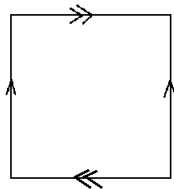
**Torus:**



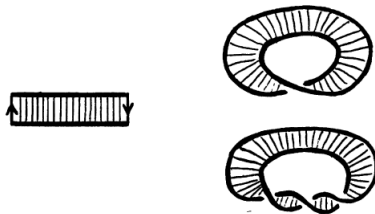
# More surfaces



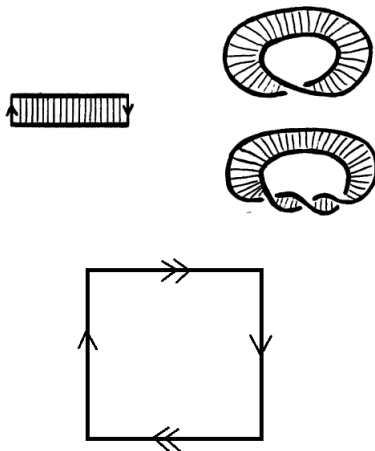
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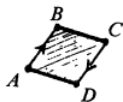
# Möbius band and projective plane



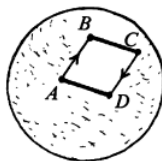
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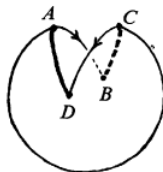
# Projective plane



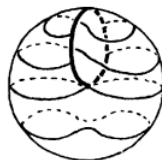
(a)



(b)

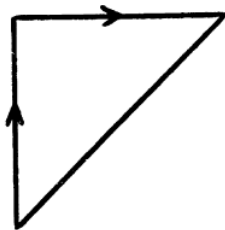


(c)

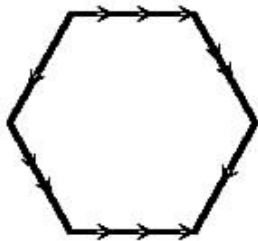


(d)

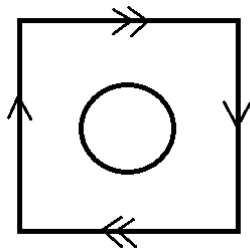
# Which surface is that?



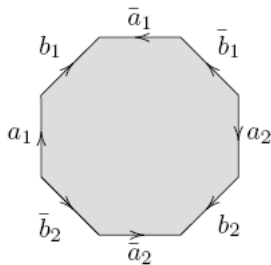
# What about this one?



The same question here



# Try to guess the question



# What happens if we glue two Möbius bands together?